Conics in Polar Form

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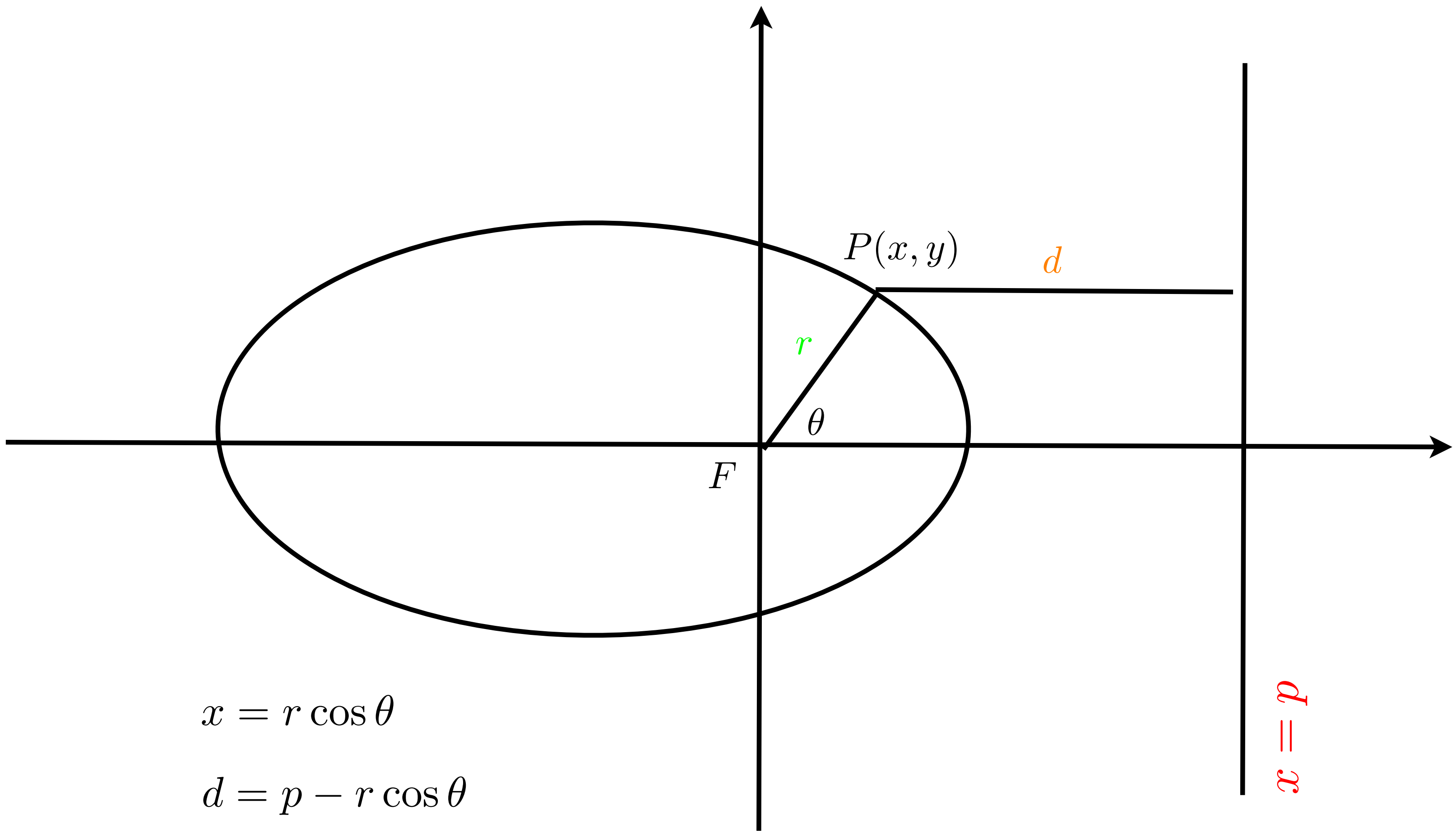
Set of all points where the ratio of the distance between a focus and a directrix is constant

This constant is called e, the eccentricity

$$e = \frac{r}{d}$$

$$r = de$$

$$d = \frac{r}{e}$$



$$x = r\cos\theta$$

$$d = p - r\cos\theta$$

$$\frac{r}{e} = p - r\cos\theta$$

$$r = ep - er\cos\theta$$

$$r + er\cos\theta = ep$$

$$r(1 + e\cos\theta) = ep$$

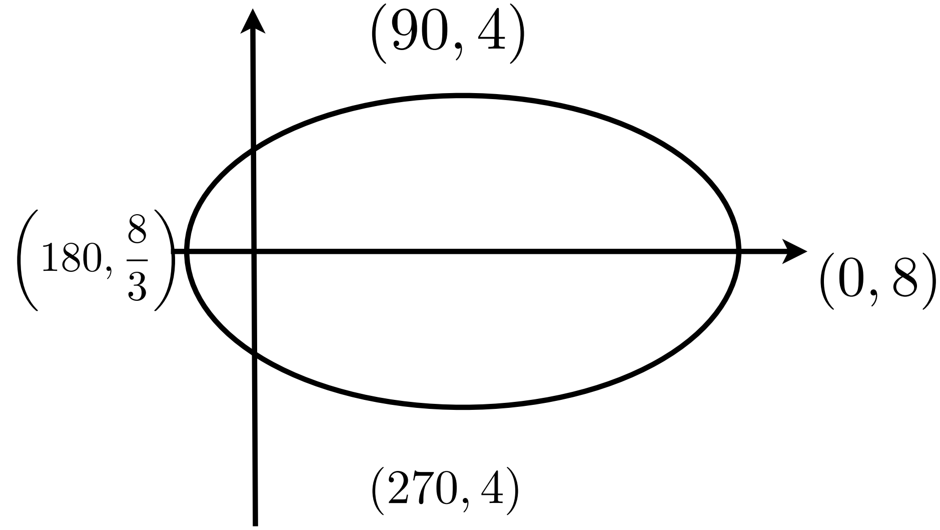
$$r = \frac{ep}{1 + e\cos\theta}$$

if the major axis is vertical...

$$r = \frac{ep}{1 + e\sin\theta}$$

Consider

$$r = \frac{8}{2 - \cos\theta}$$



If e=1, it's a parabola

If e <1, it's an ellipse

If e >1, it's a hyperbola

If c = distance from the center to the focus
and a = distance from center to a vertex

$$e = \frac{c}{a}$$

Since a is on the major axis, as before...

$$b = \sqrt{a^2 - c^2}$$

where b = distance of the perpendicular
(vertices for and ellipse, or "box" of hyerbola

$$r = \frac{8}{2 - \cos\theta}$$

Find e- get it in the form $r = \frac{ep}{1 + e\cos\theta}$

$$r = \frac{4}{1 - \frac{1}{2}\cos\theta}$$

$so \;\; e = -\frac{1}{2} \;\; an\ ellipse$

$ep = 4$

$p = -8 \;\; so\ the\ directrix\ is \;\; x = -8$

Find vertices on major axis

$if \;\; \theta = 0, \;\; r = 8$ $\qquad$ $if \;\; \theta = \pi, \;\; r = \frac{8}{3}$

$(8, 0)$ $\qquad$ $\left(-\frac{8}{3}, 0\right)$

Center at midpoint $\left(\frac{8}{3}, 0\right)$

a=distance from center to vertex: $8 - \frac{8}{3} = \frac{16}{3}$

Since $e = \frac{c}{a}, c = ae = \left(\frac{16}{3}\right)\left(-\frac{1}{2}\right) = -\frac{8}{3}$

$$b^2 = a^2 - c^2 = \frac{256}{9} - \frac{64}{9} = \frac{64}{3}$$

Converting to rectangular coords

$$r = \frac{8}{2 - \cos\theta}$$

$$2r - r\cos\theta = 8$$

$$2\sqrt{x^2 + y^2} - x = 8$$

$$(2\sqrt{x^2 + y^2})^2 = (x + 8)^2$$

$$4(x^2 + y^2) = x^2 + 16x + 64$$

$$3x^2 - 16x + 4y^2 = 64$$

$$3\left(x^2 - \frac{16}{3}x + \frac{8^2}{3^2}\right) + 4y^2 = 64 + 3\left(\frac{8^2}{3^2}\right)$$

$$3\left(x - \frac{8}{3}\right)^2 + 4y^2 = \frac{256}{3}$$

$$\frac{\left(x - \frac{8}{3}\right)^2}{\frac{256}{9}} + \frac{y^2}{\frac{256}{12}} = 1$$

or

$$\frac{\left(x - \frac{8}{3}\right)^2}{\frac{256}{9}} + \frac{4y^2}{\frac{64}{3}} = 1$$